Exercises 8

1. Use Monte Carlo simulation to estimate the value of the following integrals:

   a) $\int_1^2 x^3 + 1 dx$

   b) $\int_{-1}^1 \sqrt{1 - x^2} dx$

   c) $\int_0^\pi \left|\sin\left(x^2\right)\right| dx$

   d) $\int_0^1 x \ln\left(\sqrt{x}\right) dx$

2. Use Monte Carlo simulation to find the probability, when four dice are thrown simultaneously, that they can be split into two pairs with equal sums. For example, the roll $(3, 1, 3, 5)$ can be split into two pairs with equal sums: $3 + 3 = 1 + 5$, however $(2, 2, 2, 4)$ cannot.

3. In North Wales a child write out the name of their town in fridge magnets:

   `LLANFAIRPWLLGWYNGYLLGOGERYCHWYRNDROBWLLLLANTYSILIOGOGOGOCH`

   Every morning the child slams the fridge quite boisteriously, and there is a probability 0.85 that a letter will fall off. The fallen latter is not replaced. We are interested in N, the number of days it takes for all the `L` letters fall off. Use Monte Carlo simulation to find out the following questions:

   a) What is the expected value of N?

   b) Plot the pmf of N as a historgram.

   c) What is the probability that $N < 54$?

   d) What is the probability that $N > 70$?

4. The following code simulates a one dimentional random walk, where at each step there is a probability 0.5 of taking a step forward, and a probability 0.5 of taking a step backwards:

```
>>> def random_walk(n_steps):
...     location = 0
...     for i in range(n_steps):
...         location += random.choice([-1, 1])
...     return location
```

   a) Running many trials, find the expected distance away from the origin after 500 steps.

   b) Adapt the code so that it is a 2-dimensional random walk, where there is a probability 0.25 each of taking a step forward, backwards, to the left, and to the right. Consider the Euclidean distance from the origin.