# Problem Sheet 4

1. Use Monte Carlo simulation to estimate the value of the following integral, giving a confidence interval for your estimate.

$$I = \int_{3/2}^{5/2} x^3 + 1 \, dx$$

2. Use Monte Carlo simulation to estimate the value of the following integral, giving a confidence interval for your estimate.

$$I = \int_{-1}^{1} \sqrt{1 - x^2} \, dx$$

3. Use Monte Carlo simulation to find the probability $p$, that when four dice are thrown simultaneously, they can be split into two pairs with equal sums. For example, the roll $(3, 1, 3, 5)$ can be split into two pairs with equal sums: $3 + 3 = 1 + 5$, however $(2, 2, 2, 4)$ cannot. Give a confidence interval for your estimate.

4. The system to order food at a cafe in the centre of Cardiff behaves as a network of queues. There are three counters: the cold food counter, the hot food counter, and the till where customers pay for their food. If customers want cold food only, they arrive and join the cold food till; if they want hot food only, they arrive and join the hot food till; if they want both cold and hot food they must first queue at the cold food counter, and then at the hot food counter. After picking up their food, they must then queue at the till to pay.

   - Customers arrive at the cold food counter at a rate of 18 per hour,
   - Customers arrive at the hot food counter at a rate of 12 per hour,
   - 30% of customers who queue at the cold food counter also want hot food,
   - On average it takes 1 minute to be served cold food, 2 and a half minutes to be served hot food, and 2 minutes to pay,
   - There is 1 server at the cold food counter, 2 servers at the hot food counter, and 2 servers at the till,
   - The cafe is open for 3 hours during the lunch hour.

   The cafe would like to know how many customers they expect to be served each lunchtime on average. Perform a discrete-event simulation in Ciw, and show the conceptual model.

5. In an A&E department that is open for 24 hours, patients arrive according to a Poisson process at rate 35 per hour. Of these, $^1/_7$th are categorised as Triage category 1; $^2/_7$th are categorised as Triage category 2; and $^4/_7$the are categorised at Triage category 3. Patients wait to be seen by one of the 18 doctors on duty at the A&E, and they are seen by a doctor according to priorities: Triage 1 patients have priority over Triage 2 patients, who in turn have priority over Triage 3 patients. The time it takes to see each patient depends on their triage category: Triage 3 patients are given a 15 minute appointment; Triage 2 patients' time with a doctor is Uniformly distributed between 15 and 25 minutes; and Triage 1 patients' time with a doctor is Uniformly distributed between 20 and 90 minutes. What is the average waiting time for patients of each triage category?