

Problem Sheet 6

1. Assume that the costs $c_1, c_2, c_3, c_4, c_5, c_6 > 0$ and all distinct. Explain why the solution below cannot be optimal.

	X	Y	Z	
A	30 c_1	c_2	70 c_3	100
B	30 c_4	80 c_5	40 c_6	150
	60	80	110	250

2. Consider the following transportation problem, with four sources A, B, C and D , and four destinations W, X, Y and Z :

	W	X	Y	Z	
A	6	3	10	2	5
B	1	14	7	4	19
C	5	9	8	12	6
D	2	11	15	13	5
	7	13	11	4	35

- How many basic and non-basic variables does this problem have?
- Find a feasible solution using the North West Corner method. What would be the cost of using this solution?
- Find a feasible solution using the Minimum Cost method. What would be the cost of using this solution?

3. ToyAutos is a small car manufacturer based in Wales, producing 100 vehicles per year. They have three factories, one in Barry producing 35 vehicles a year, one in Port Talbot producing 45 vehicles a year, and another in Wrexham producing 20 vehicles a year. They have three dealerships, one in Bangor selling 15 cars a year, one in Cardiff selling 70 cars a year, and another in Newport selling 15 cars a year.

According to Google Maps, the distances between each of the factories and dealerships is as follows:

From	To	Miles
Barry	Bangor	188
Barry	Cardiff	10
Barry	Newport	25
Port Talbot	Bangor	165
Port Talbot	Cardiff	36
Port Talbot	Newport	42
Wrexham	Bangor	64
Wrexham	Cardiff	140
Wrexham	Newport	126

ToyAutos would like to know how many cars to send to each dealership from each factory, to satisfy the supply and demand in each location, and to minimise the total number of miles travelled.

(Hint: begin by producing an initial feasible solution with the North West Corner method, then use the Stepping-Stone algorithm)

4. Consider the following transportation problem:

	X	Y	Z	
A	7	7	4	20
B	3	6	5	40
C	6	9	2	20
	30	10	40	80

- (a) Find a feasible solution using the Minimum Cost method.
- (b) The problem is degenerate. Identify a basic variable that is set to zero.
- (c) Find an optimal solution using the stepping stone algorithm.
5. A small fire department in rural Galicia runs five fire stations, F_1 , F_2 , F_3 , F_4 and F_5 . These fire stations house 3, 3, 1, 10, and 5 fire engines respectively. Two calls come in simultaneously: there have been two major fire disasters, D_1 and D_2 . The department estimates that D_1 will require 14 fire engines, and D_2 will require 8 fire engines. The time (in minutes) from each fire station to the location of each disaster is given by:

	F_1	F_2	F_3	F_4	F_5
D_1	8	6	3	10	5
D_2	3	9	2	7	4

By first using the minimum cost method, and then the stepping stone method, devise a plan for which fire engines should be dispatched to which disaster.

6. Consider the transportation problem below, with demand nodes A, B and C, and supply nodes X, Y and Z. First use the minimum cost method to find a basic assignment, then use the stepping stone algorithm to show that this is the optimal assignment. Which demand nodes are unsatisfied?

	X	Y	Z	
A	5	9	2	35
B	11	8	3	15
C	15	3	5	5
	5	10	20	