Markov Chain Modelling: Exercises 1

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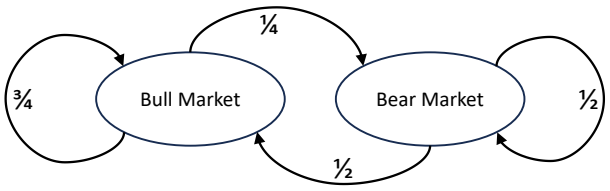
1. Consider the Markov chain defined on four states $\{A_1, A_2, A_3, A_4\}$ and transition probability matrix:

$$P = \begin{pmatrix} x & 0.5 & 0.0 & 0.0 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.1 & 0.25 & y & 0.45 \\ 0.2 & z & 0.7 & 0.1 \end{pmatrix}$$

   What values must x, y, and z take for P to be a valid transition probability matrix?

2. A country's economy can be described as either a Bull market (where stock prices rise and things are going well), or a Bear market (where stock prices fall and things are not going so well). The economy is categorised as such each quarter. This process can be described as a discrete-time Markov chain, with probabilities of being in each state in the next quarter:



   If the country is currently in a Bull market, what is the probability of being in either a Bull or a Bear marker in three quarters times?

3. You own a banana farm. Each month, the banana farm can either produce a good yield, a bad yield, or no yield:

   - On a month where a good yield is produced, you can harvest the bananas and sell them for a good price. Once harvested, there are no bananas left.
   - On a month where a bad yield is produced, you do not harvest, and wait until a better yield is produced.
   - On a month with no yield, it is impossible to harvest, and so again you must wait.

   Therefore you always wait for a good yield of bananas.

   It takes one month to grow a bad yield from no yield; and it takes one month to grow a good yield from a bad yield.

   However, you have a monkey problem! Each month there is a probability of 0.1 that the monkeys eat your newly grown bananas. Therefore:

   - there is a probability 0.1 that a bad yield remains bad after a month;
   - there is a probability 0.1 that no yield remains without any yield after a month;
   - and a probability 0.1 that the month after a good yield the farm produces no yield.

   Draw the Markov chain for this scenario, and give the transition probability matrix.

   This month you observed a bad yield. What is the probability of having a good yeild and therefore making a profit in:

   (a) 1 month's time?

   (b) 2 month's time?

   (c) 3 month's time?