

Markov Chain Modelling: Exercises 2

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1. Consider the banana farm scenario from earlier.
 - (a) What is the steady-state distribution?
 - (b) In steady-state, what is the probability of making a profit?

2. A mental health doctor is trying to understand a patient's mental state. They ask the patient to record daily whether they feel Calm, Mildly Anxious, or Very Anxious. Crunching the data the doctor finds:
 - On a calm day, $\frac{1}{3}$ of the time they will remain calm tomorrow, and $\frac{2}{3}$ of the time they will become mildly anxious tomorrow;
 - On a mildly anxious day, $\frac{1}{4}$ of the time they will become calm tomorrow, $\frac{1}{2}$ the time they remain mildly anxious tomorrow, while $\frac{1}{4}$ of the time they become very anxious tomorrow;
 - On a very anxious day, only $\frac{1}{5}$ of the time will they become calm tomorrow, $\frac{2}{5}$ of the time they will become mildly anxious, however $\frac{2}{5}$ of the time they remain very anxious tomorrow.
 - (a) Draw the discrete-time Markov chain describing the patient's mental state.
 - (b) Find the steady-state probabilities.
 - (c) The doctor devises a medication plan: on calm days the patient should not take any medication; on mildly anxious days they should take a pill of type A, costing $1p$ per pill; and on very anxious days they should take a pill of type B, costing $23p$ per pill. What is the expected yearly cost for this medication plan?