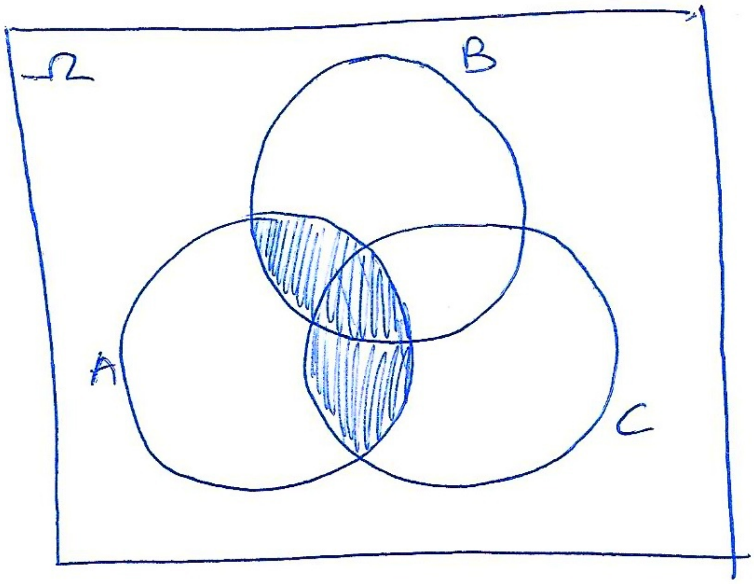
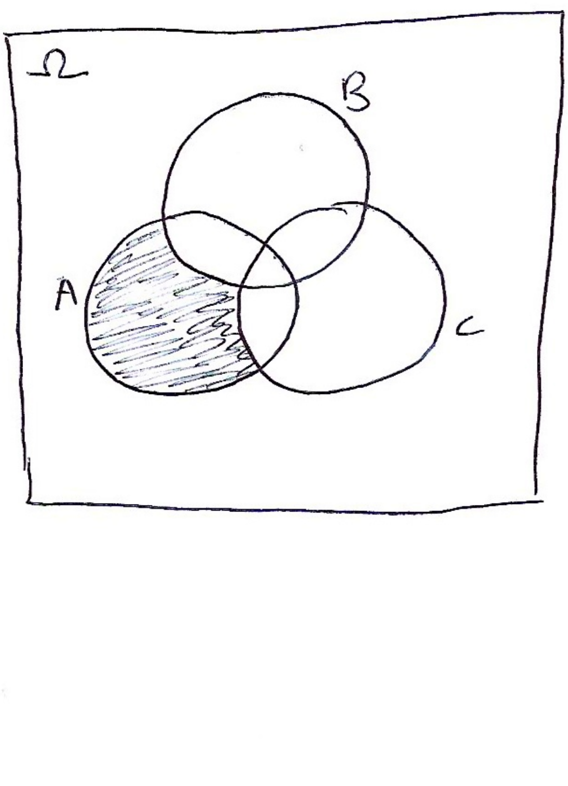
Week 1 - Example Excercises

1) Shade in $(A \cap B) \cup (A \cap C)$.



Note that this is the same as $A \cap (B \cup C)$, due to the distributive law.

2) What set is shaded in?



$A \cap (\overline{B \cup C})$

or

$A \cap (\bar{B} \cap \bar{C})$

Note that these are the same due to deMorgan's laws.

3) In the Museo del Prado in Madrid there are 7600 paintings. 5000 are by Spanish artists, and 3800 have Spanish subjects.

- give upper and lower bounds for the number of "truely Spanish" paintings (both artist and subject ar Spanish).

Let A = the set of paintings with Spanish artists

Let B = the set of paintings with Spanish subjects

Now $|A| = 5000$, $|B| = 3800$, $|\Omega| = 7600$.

Consider $A \cap S$, the set of "truely Spanish" paintings,

- Worst case is when A and S "overlap" (intersect) by as little as possible, so $A \cup S = \Omega$.

$$\begin{aligned} |A \cap S| &= |A| + |B| - |A \cup S| \\ &= |A| + |B| - |\Omega| \\ &= 5000 + 3800 - 7600 \\ &= 1200. \end{aligned}$$

- Best case is when A and S completely overlap, e.g. $S \subset A$, so $A \cap S = S$.

$|A \cap S| = |S| = 3800.$

$$\therefore \quad 1200 \le |A \cap S| \le 3800$$

- give upper and lower bounds for the number of "truely foreign" paintings (neither artist nor subject is Spanish).

Now consider $\overline{A \cup S}$, the set of "truely foreign" paintings.

- Worst case is when A and S take up as much of Ω as possible. As $|A| + |S| \geq |\Omega|$, we can have that

  $A \cup S = \Omega$.

  $$\begin{aligned} |\overline{A \cup S}| &= |\Omega| - |A \cup S| \\ &= |\Omega| - |\Omega| \\ &= 0 \end{aligned}$$

- Best case when A and S take up as little of Ω as possible, e.g. $S \subset A$.

  $$\begin{aligned} |\overline{A \cup S}| &= |\Omega| - |A \cup S| \\ &= |\Omega| - (|A| + |S| - |A \cap S|) \\ &= |\Omega| - |A| \\ &= 7600 - 500 \\ &= 2600. \end{aligned}$$

$\therefore$ $$0 \leq |\overline{A \cup S}| \leq 2600$$

4) State and show whether the following are true or false:

- $A \cup \overline{B} = B \cup \overline{A}$

False.
Can be shown with a counter-example:
Let A = set of even numbers
Let B = set of prime numbers

$A \cup \overline{B} = \{2, 4, 6, 8, 9, 10, 12, 14, 15, \ldots\}$
$B \cup \overline{A} = \{1, 2, 3, 5, 7, 9, \ldots\}$

$\therefore$ They are not equal.

- If $B \in P(A)$, the power set of A, then $B \subseteq A$.

True by definition. $P(A)$ is defined as the set of all possible subsets of A.