# WEEK 2 – EXAMPLE EXERCISES

1) $n$ die are rolled, what is the probability that the maximum is greater than 1?

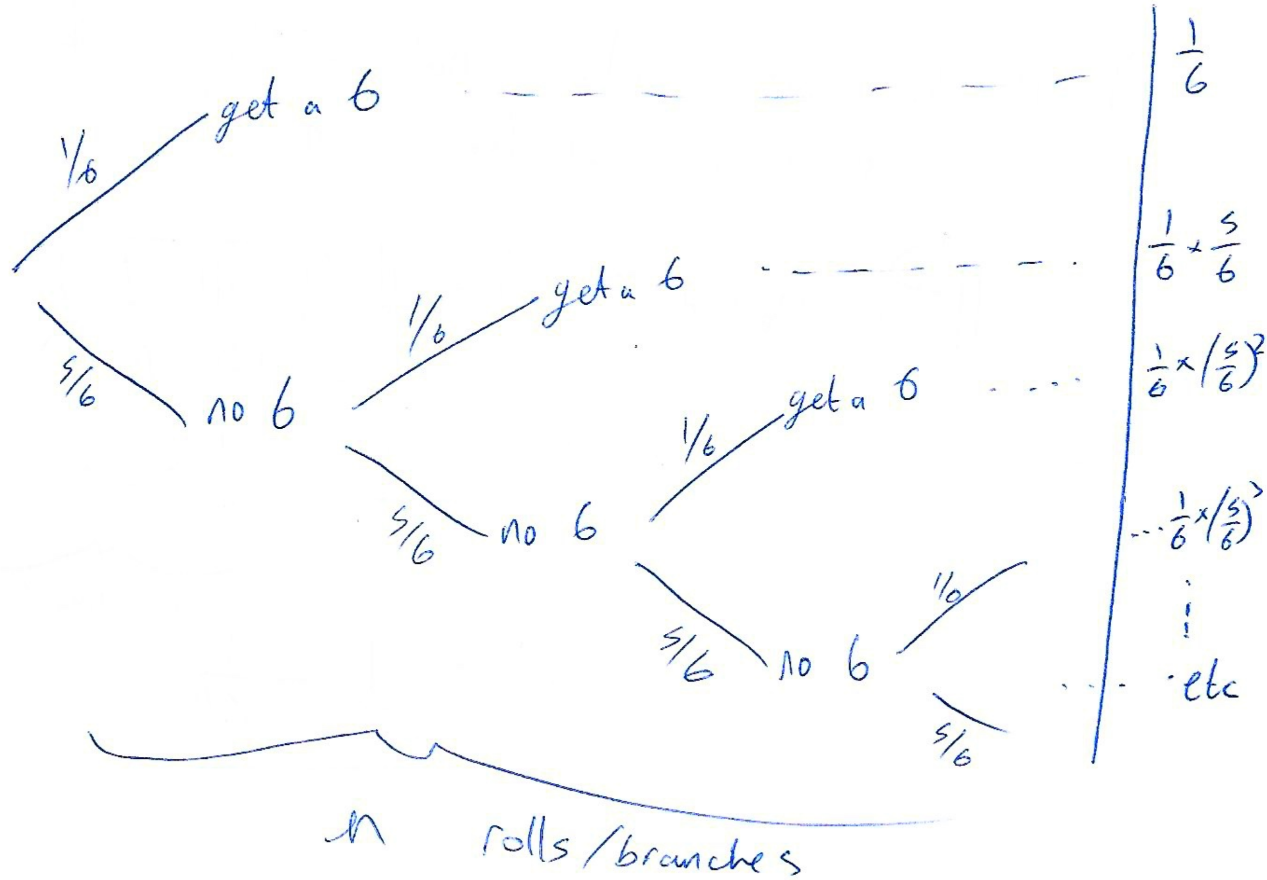
$$\mathbb{P}(\text{maximum} > 1) = \mathbb{P}(\overline{\text{all } 1})$$
$$= 1 - \mathbb{P}(\text{all } 1)$$
$$= 1 - \underbrace{\left(\tfrac{1}{6} \times \tfrac{1}{6} \times \dots \times \tfrac{1}{6}\right)}_{n \text{ times}}$$
$$= 1 - \frac{1}{6^n}$$

2) $n$ die are rolled, what is the probability that the maximum value is a 6?

There are two equivalent ways to answer this:

- $$\mathbb{P}(\text{max value} = 6) = \mathbb{P}(\text{at least one } 6)$$
$$= \mathbb{P}(\overline{\text{no sixes}})$$
$$= 1 - \mathbb{P}(\text{no sixes})$$
$$= 1 - \underbrace{\left(\tfrac{5}{6} \times \tfrac{5}{6} \times \dots \times \tfrac{5}{6}\right)}_{n \text{ times}}$$
$$= 1 - \left(\tfrac{5}{6}\right)^n$$

• Or using a tree diagram:




$$\mathbb{P}(\text{max value}=6) = \mathbb{P}(\text{at least 1 six})$$

$$= \mathbb{P}(6 \text{ on first}) + \mathbb{P}(6 \text{ on second}) + \mathbb{P}(6 \text{ on } 3^{rd}) + \dots$$

$$= \frac{1}{6} + \left(\frac{1}{6}\times\frac{5}{6}\right) + \left(\frac{1}{6}\times\left(\frac{5}{6}\right)^2\right) + \left(\frac{1}{6}\times\left(\frac{5}{6}\right)^3\right) + \dots + \left(\frac{1}{6}\times\left(\frac{5}{6}\right)^{n-1}\right)$$

$$= \frac{1}{6}\left(1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \dots + \left(\frac{5}{6}\right)^{n-1}\right)$$

$$= \frac{1}{6}\left(\frac{1-\left(\frac{5}{6}\right)^n}{1-5/6}\right)$$

$$\boxed{\text{as } \quad a + ar + ar^2 + \dots + ar^n = a\left(\frac{1-r^{n+1}}{1-r}\right)}$$

$$= 1 - \left(\frac{5}{6}\right)^n$$

3) 80% of my junk emails contain the word "sale".
10% of my desired emails contain the word "sale"
30% of my emails are junk.
An email arrives containing the word "sale", what is the probability that it is junk?

Let $J$ = an email is junk, and $S$ = an email contains "sale".

we know:

$$\mathbb{P}(S|J) = 0.8$$
$$\mathbb{P}(S|\bar{J}) = 0.1$$
$$\mathbb{P}(J) = 0.3$$

we want $\mathbb{P}(J|S)$.

from Bayes' theorem: $\mathbb{P}(J|S) = \frac{\mathbb{P}(S|J)\mathbb{P}(J)}{\mathbb{P}(S)}$

However we do not know $\mathbb{P}(S)$, but we can use total probability:

$$\begin{aligned}\mathbb{P}(S) &= \mathbb{P}(S|J)\mathbb{P}(J) + \mathbb{P}(S|\bar{J})\mathbb{P}(\bar{J}) \\ &= (0.8)(0.3) + (0.1)(1-0.3) \\ &= 0.31\end{aligned}$$

$$\therefore \mathbb{P}(J|S) = \frac{\mathbb{P}(S|J)\mathbb{P}(J)}{\mathbb{P}(S)} = \frac{0.8 \times 0.3}{0.31}$$

$$= 0.7742$$