Deadlock in Queueing Networks

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CORS 2016 - Banff

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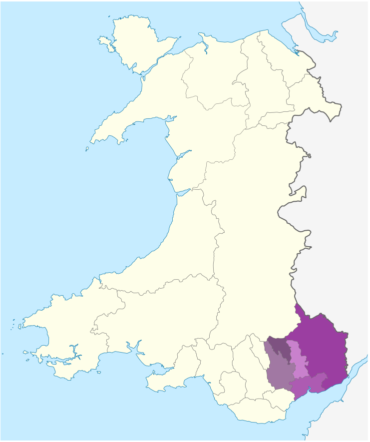
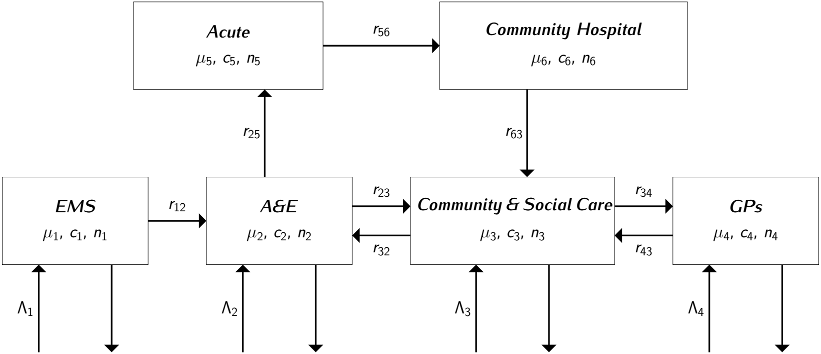
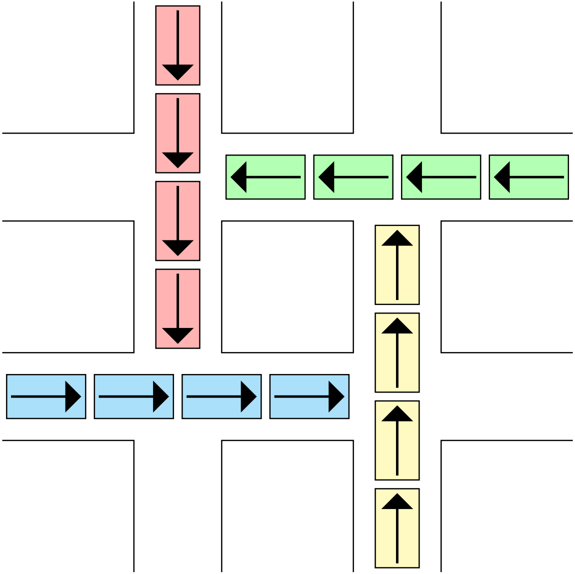
Health Board

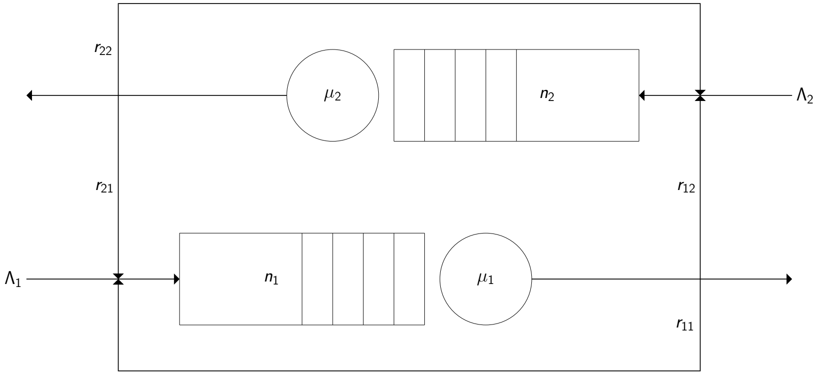


Elderly People's Flows Through Health System

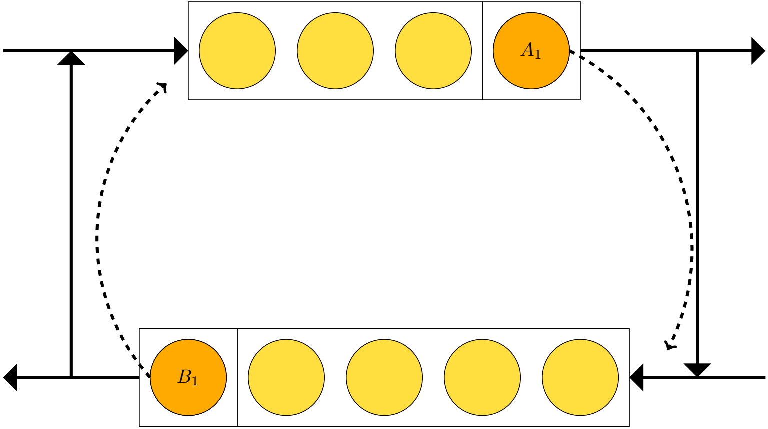


Deadlock

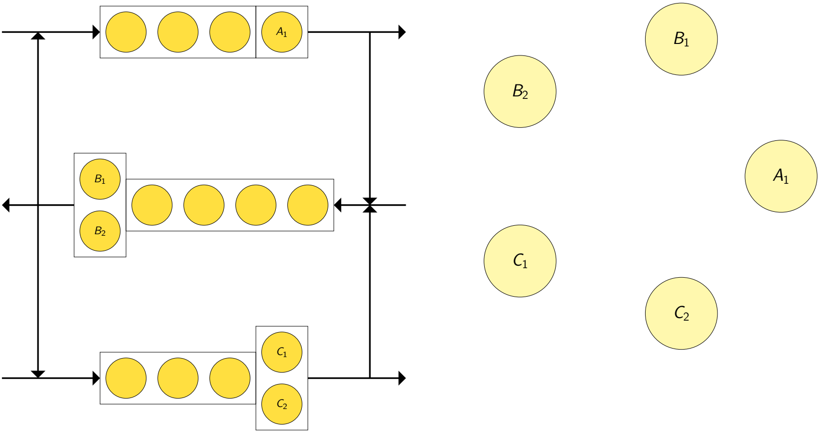


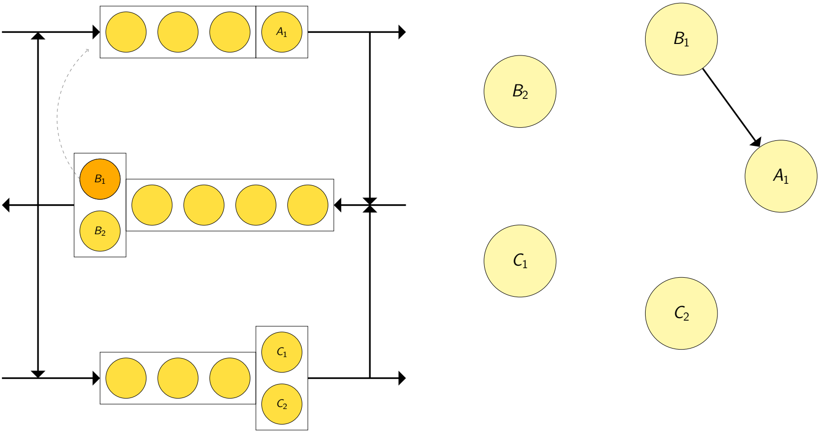


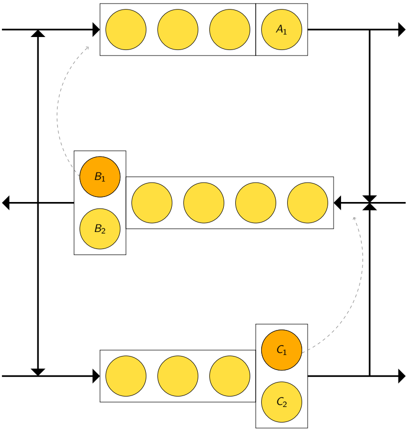
r_{22}
μ_2
n_2
Λ_2
r_{21}
r_{12}
Λ_1
n_1
μ_1
r_{11}



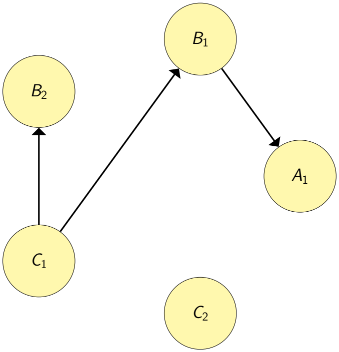
A_1
B_1


A_1
B_1
B_2
C_1
C_2
B_1
B_2
A_1
C_1
C_2

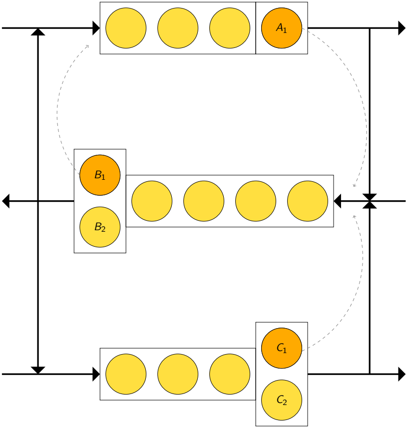

A1
B1
B2
C1
C2
B1
B2
A1
C1
C2

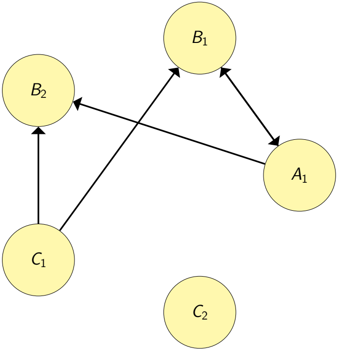


A_1
B_1
B_2
C_1
C_2



B_1
B_2
A_1
C_1
C_2


A_1
B_1
B_2
C_1
C_2


B_1
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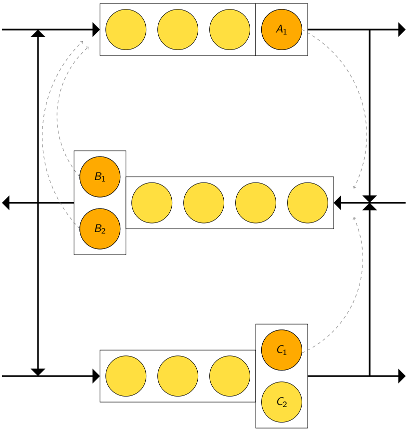
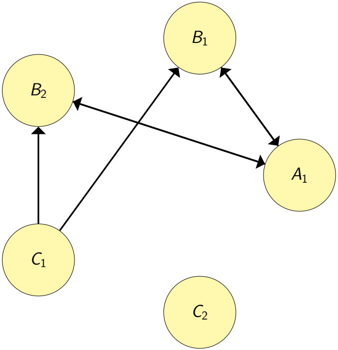
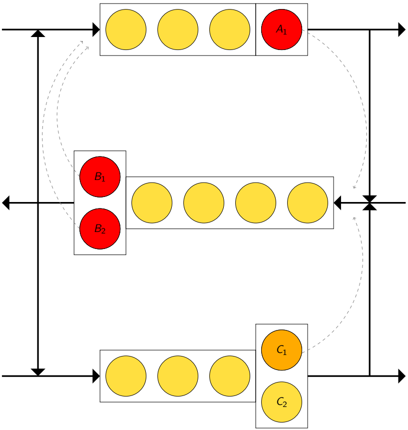
A_1
C_1
C_2



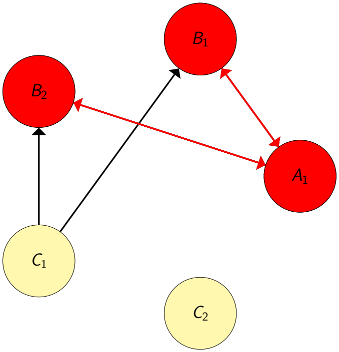
A_1
B_1
B_2
C_1
C_2



B_1
B_2
A_1
C_1
C_2

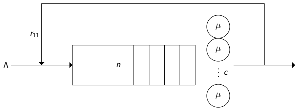


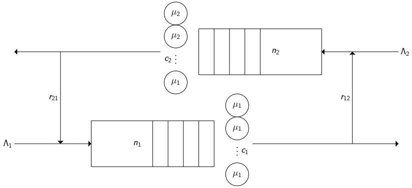
A_1
B_1
B_2
C_1
C_2

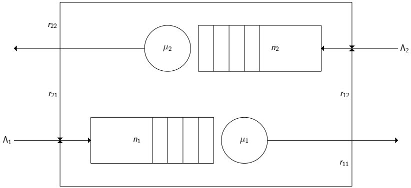


B_1
B_2
A_1
C_1
C_2

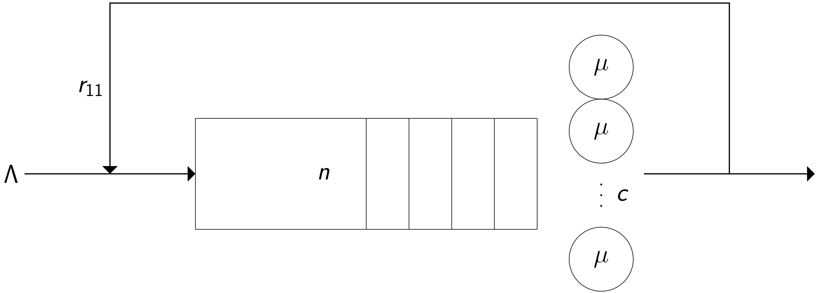
Three Deadlocking Queueing Networks







Markovian Model of Deadlock



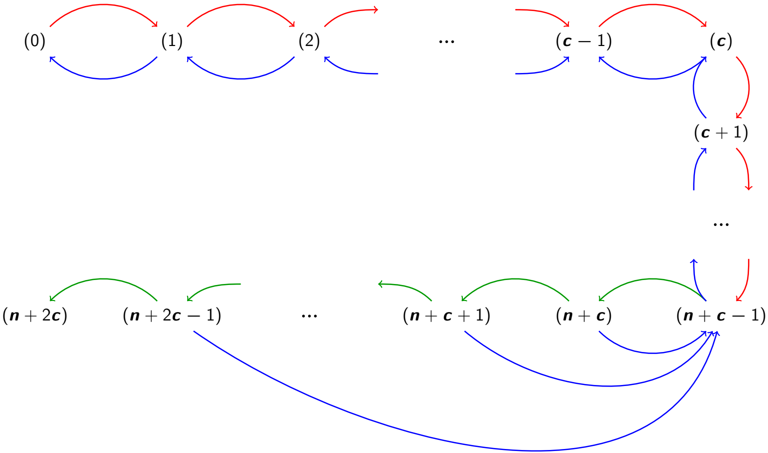
(i)

$$S = \{i \in \mathbb{N} \mid 0 \leq i \leq n + 2c\}$$

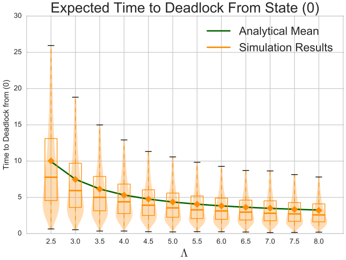
Define $\delta = i_2 - i_1$

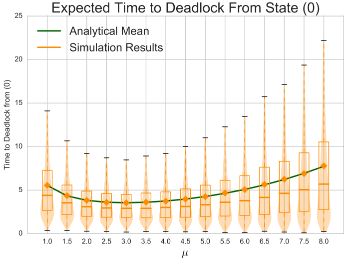
$$q_{i_1,i_2} = \left\{ \begin{array}{rr} \Lambda & \text{if } \delta = 1 \\ (1 - r_{11})\mu \min(i, c) & \text{if } \delta = -1 \\ 0 & \text{otherwise} \end{array} \right\} \quad \text{if } i_1 < n + c$$

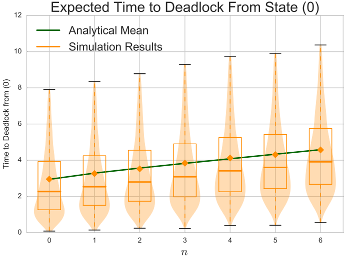
$$q_{i_1,i_2} = \left\{ \begin{array}{rr} (c - b)r_{11}\mu & \text{if } \delta = 1 \\ (1 - r_{11})(c - b)\mu & \text{if } \delta = -b - 1 \\ 0 & \text{otherwise} \end{array} \right\} \quad \text{if } i_1 = n + c + b \quad \forall \quad 0 \leq b \leq c$$

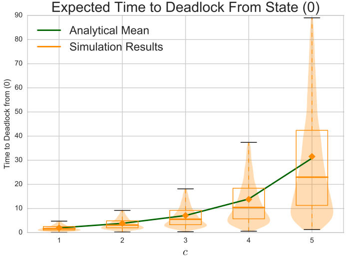

(0)
(1)
(2)
...
(c − 1)
(c)
(c + 1)
...
(n + 2c)
(n + 2c − 1)
...
(n + c + 1)
(n + c)
(n + c − 1)

Times to Deadlock

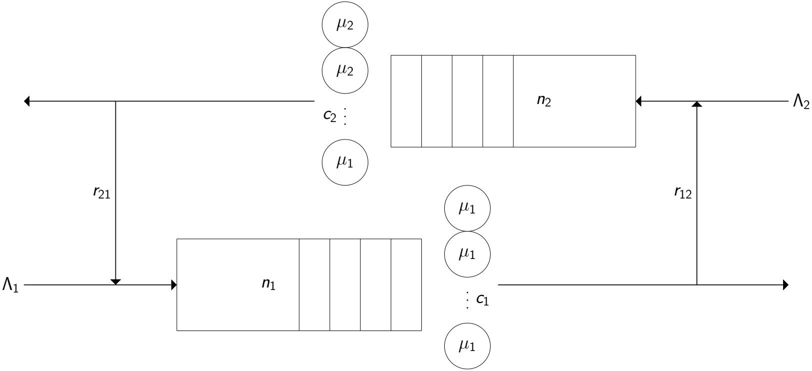








Markovian Model of Deadlock

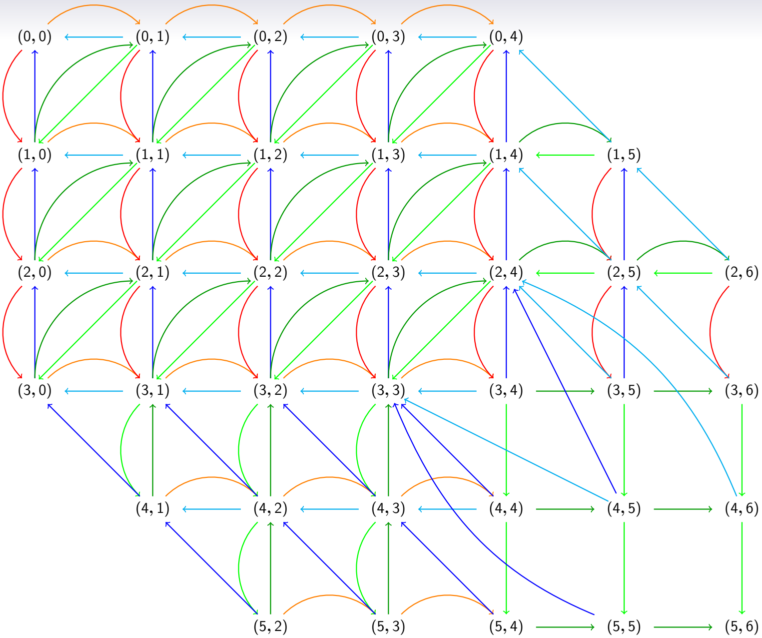


$$(i, j)$$

$$S = \{(i,j) \in \mathbb{N}^{(n_1+c_1+c_2)\times(n_2+c_2+c_1)} \mid i \leq n_1 + c_1 + j,\ j \leq n_2 + c_2 + i\}$$

$$\begin{aligned}
\delta &= (i_2, j_2) - (i_1, j_1) \\
b_1 &= \max(0, i_1 - n_1 - c_1) \\
b_2 &= \max(0, i_2 - n_2 - c_2) \\
s_1 &= \min(i_1, c_1) - b_2 \\
s_2 &= \min(i_2, c_2) - b_1
\end{aligned}$$

	$j_1 < n_2 + c_2$	$j_1 = n_2 + c_2$	$j_1 > n_2 + c_2$
$i_1 < n_1 + c_1$	Λ_1 if $\delta = (1,0)$ Λ_2 if $\delta = (0,1)$ $r_{12}s_1\mu_1$ if $\delta = (-1,1)$ $r_{21}s_2\mu_2$ if $\delta = (1,-1)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-1,0)$ $(1-r_{21})s_2\mu_2$ if $\delta = (0,-1)$	Λ_1 if $\delta = (1,0)$ $r_{12}s_1\mu_1$ if $\delta = (0,1)$ $r_{21}s_2\mu_2$ if $\delta = (1,-1)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-1,0)$ $(1-r_{21})s_2\mu_2$ if $\delta = (0,-1)$	Λ_1 if $\delta = (1,0)$ $r_{12}s_1\mu_1$ if $\delta = (0,1)$ $r_{21}s_2\mu_2$ if $\delta = (0,-1)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-1,0)$ $(1-r_{21})s_2\mu_2$ if $\delta = (-1,-1)$
$i_1 = n_1 + c_1$	Λ_2 if $\delta = (0,1)$ $r_{12}s_1\mu_1$ if $\delta = (-1,1)$ $r_{21}s_2\mu_2$ if $\delta = (1,0)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-1,0)$ $(1-r_{21})s_2\mu_2$ if $\delta = (0,-1)$	$r_{12}s_1\mu_1$ if $\delta = (0,1)$ $r_{21}s_2\mu_2$ if $\delta = (1,0)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-1,0)$ $(1-r_{21})s_2\mu_2$ if $\delta = (0,-1)$	$r_{12}s_1\mu_1$ if $\delta = (0,1)$ $r_{21}s_2\mu_2$ if $\delta = (1,0)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-1,0)$ $(1-r_{21})s_2\mu_2$ if $\delta = (-1,-1)$
$i_1 > n_1 + c_1$	Λ_2 if $\delta = (0,1)$ $r_{12}s_1\mu_1$ if $\delta = (-1,0)$ $r_{21}s_2\mu_2$ if $\delta = (1,0)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-1,-1)$ $(1-r_{21})s_2\mu_2$ if $\delta = (0,-1)$	$r_{12}s_1\mu_1$ if $\delta = (0,1)$ $r_{21}s_2\mu_2$ if $\delta = (1,0)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-1,-1)$ $(1-r_{21})s_2\mu_2$ if $\delta = (0,-1)$	$r_{12}s_1\mu_1$ if $\delta = (0,1)$ $r_{21}s_2\mu_2$ if $\delta = (1,0)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-\min(b_1+1, b_2+1), -\min(b_1, b_2+1))$ $(1-r_{21})s_2\mu_2$ if $\delta = (-\min(b_1+1, b_2), -\min(b_1+1, b_2+1))$



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(3, 6)
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(4, 4)
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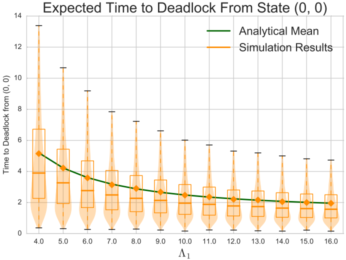
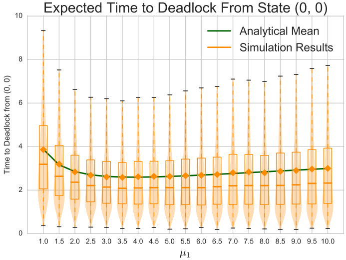
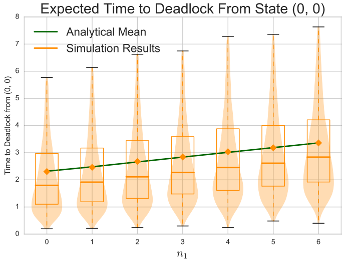
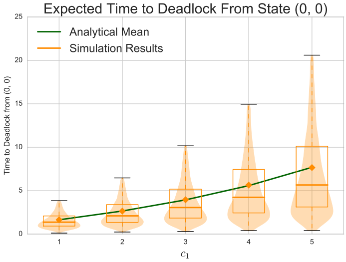
(5, 3)
(5, 4)
(5, 5)
(5, 6)

Times to Deadlock

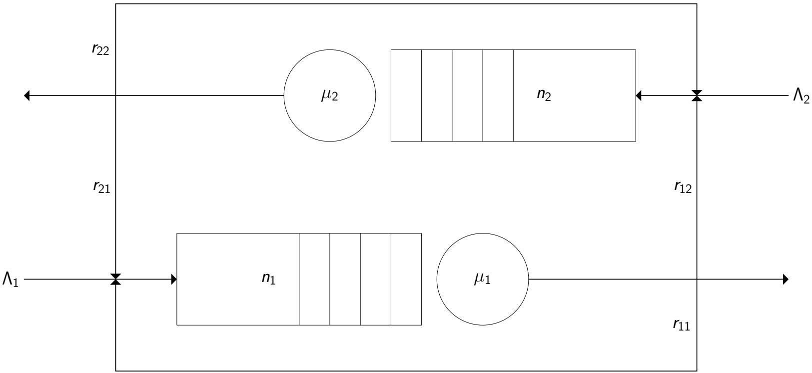








Markovian Model of Deadlock



$$(i, j)$$

$$S = \{(i,j) \in \mathbb{N}^{(n_1+2 \times n_2+2)} \mid 0 \leq i+j \leq n_1+n_2+2\} \cup \{(-1),(-2),(-3)\}$$

Define $\delta = (i_2, j_2) - (i_1, j_1)$

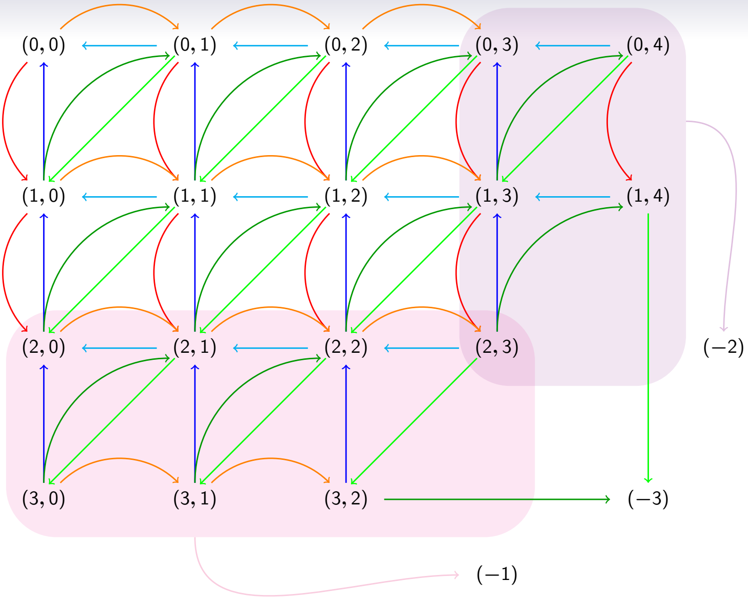
$$q_{(i_1,j_1),(i_2,j_2)} = \begin{cases} \left.\begin{matrix} \Lambda_1 & \text{if } i_1 \leq n_1 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (1,0) \\ \left.\begin{matrix} \Lambda_2 & \text{if } j_1 \leq n_2 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (0,1) \\ \left.\begin{matrix} (1-r_{12})\mu_1 & \text{if } j_1 < n_2+2 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (-1,0) \\ \left.\begin{matrix} (1-r_{21})\mu_2 & \text{if } i_1 < n_1+2 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (0,-1) \\ \left.\begin{matrix} r_{12}\mu_1 & \text{if } j_1 < n_2+2 \text{ and } (i_1,j_1) \neq (n_1+2, n_2) \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (-1,1) \\ \left.\begin{matrix} r_{21}\mu_2 & \text{if } i_1 < n_1+2 \text{ and } (i_1,j_1) \neq (n_1, n_2+2) \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (1,-1) \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1,j_1),(-1)} = \begin{cases} r_{11}\mu_1 & \text{if } i > n_1 \text{ and } j < n_2+2 \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1,j_1),(-2)} = \begin{cases} r_{22}\mu_2 & \text{if } j > n_2 \text{ and } i < n_1+2 \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1,j_1),(-3)} = \begin{cases} r_{21}\mu_2 & \text{if } (i,j) = (n_1, n_2+2) \\ r_{12}\mu_1 & \text{if } (i,j) = (n_1+2, n_2) \\ 0 & \text{otherwise} \end{cases}$$

$$q_{-1,s} = q_{-2,s} = q_{-3,s} = 0$$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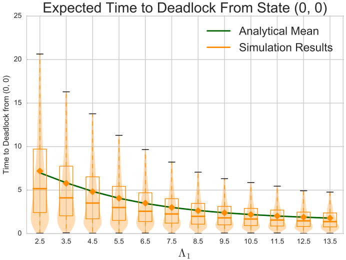
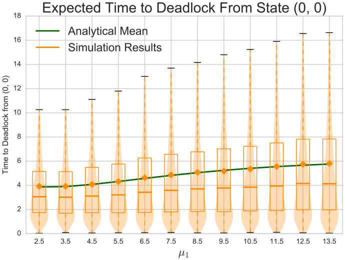
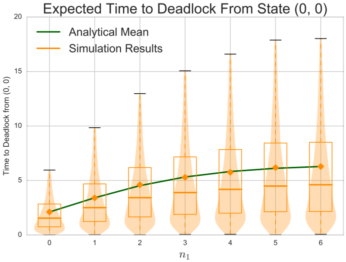
(−2)
(3, 0)
(3, 1)
(3, 2)
(−3)
(−1)

Times to Deadlock







Summary

Summary

- Investigate deadlock in open restricted queueing networks, especially the time until deadlock occurs.
- Method of detecting deadlock in discrete event simulations of queueing networks.
- Three Markov models of deadlocking queueing networks.

To Do...

- Build and parameterise patient flow networks from data.
- Use queueing network analysis and simulation to investigate impact of the OPICP.
- Determine the OPICP's effect on demand and workforce needs.

Thank You

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