Queueing Networks for a Healthcare System

Deadlocking & Reinforcement Learning

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Young OR 19 - Aston University 2015

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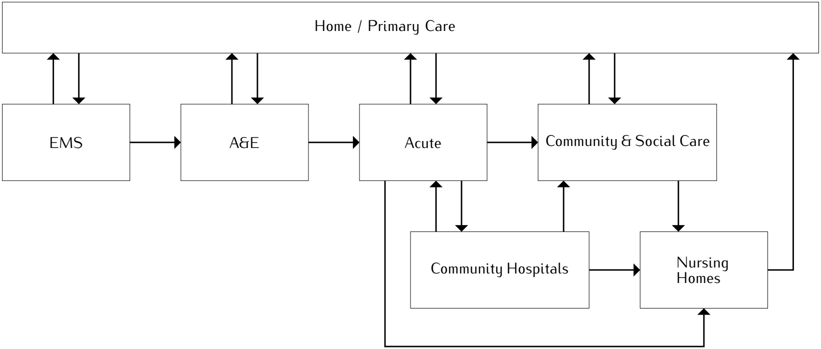
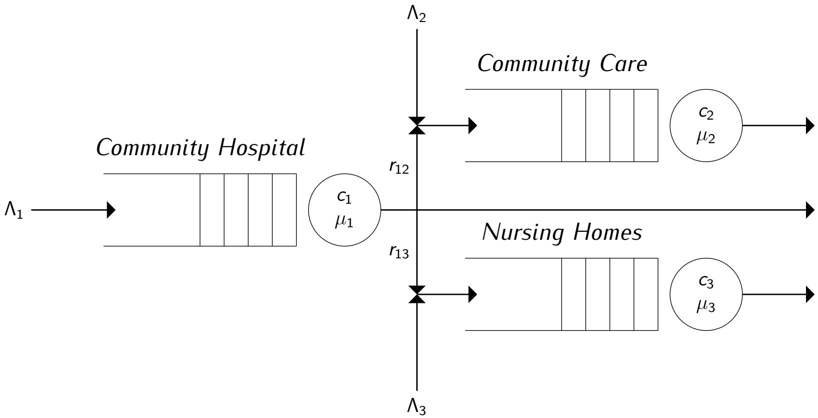
Map of Healthcare System

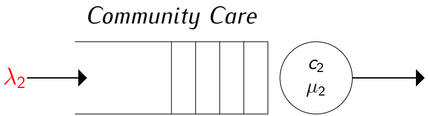


Jackson Networks



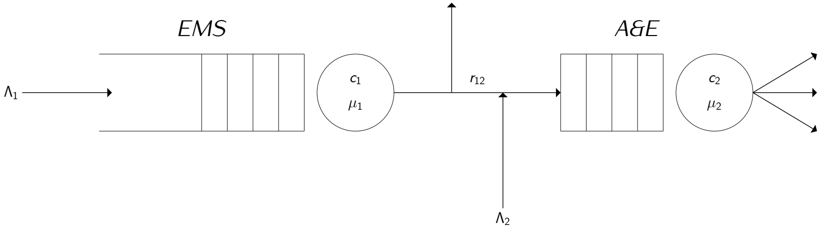
Jackson Networks

$$\lambda_i = \Lambda_i + \sum_j r_{ji} \lambda_j$$



$$P(k_1, k_2, \ldots, k_M) = \prod_{i=1}^{M} P_i(k_i)$$

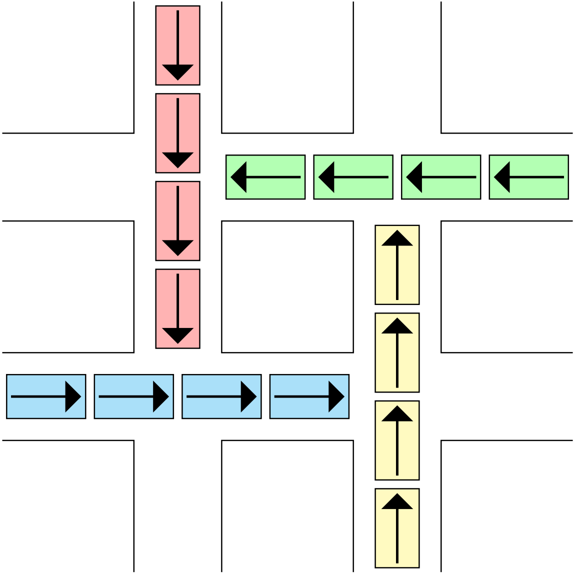
Restricted Networks

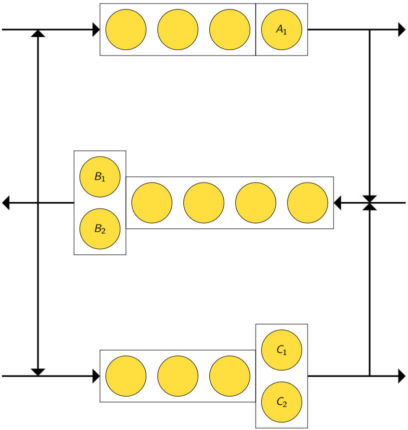


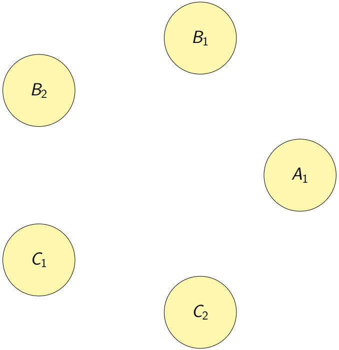
Restricted Networks

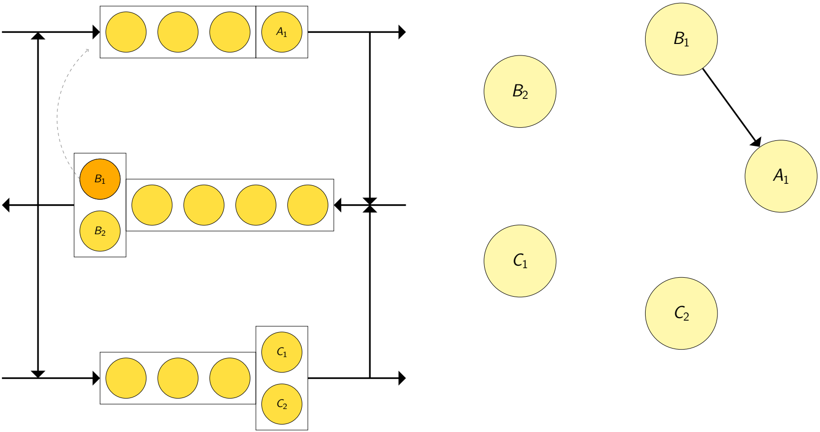
- Markov Chain Models
- Approximation Methods
- Simulation

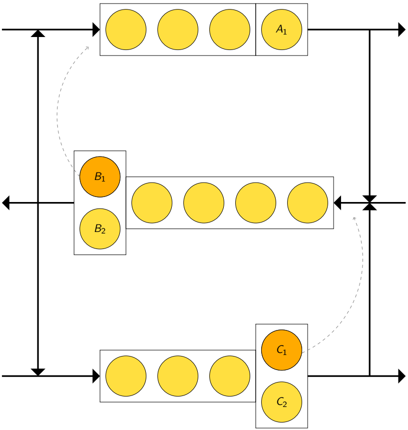
Deadlock

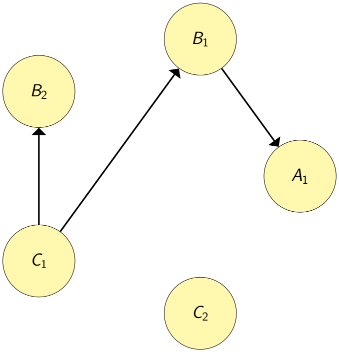


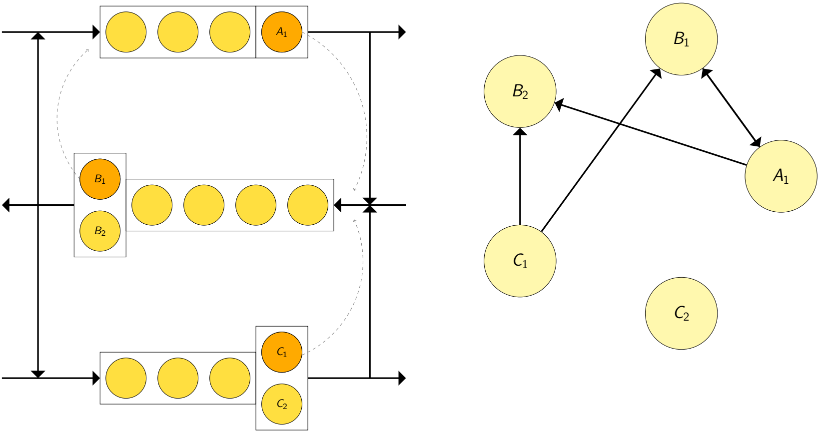

A_1
B_1
B_2
C_1
C_2


B_1
B_2
A_1
C_1
C_2

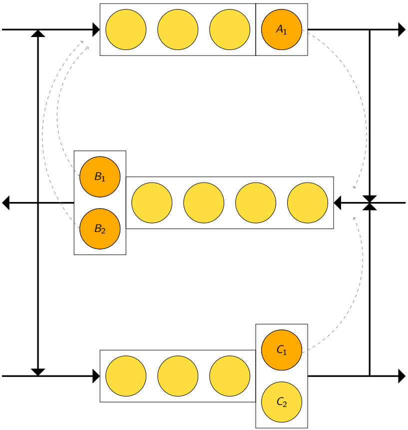

A_1
B_1
B_2
C_1
C_2
B_1
B_2
A_1
C_1
C_2

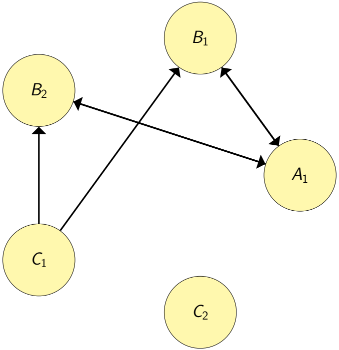

A_1
B_1
B_2
C_1
C_2

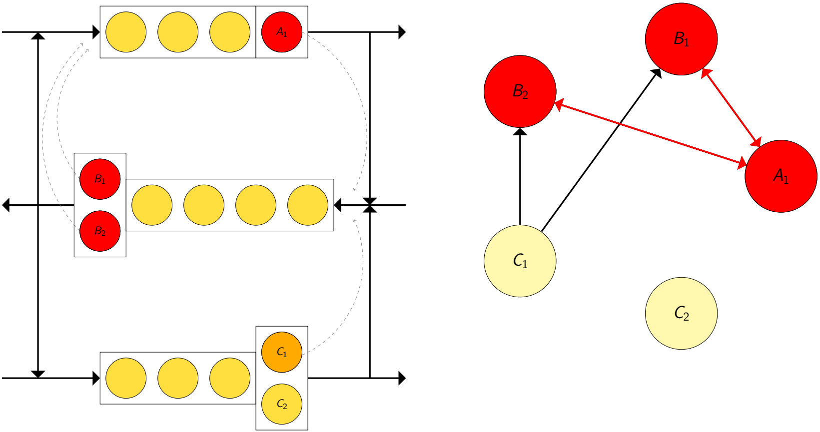

B_1
B_2
A_1
C_1
C_2



A1
B1
B2
C1
C2
B1
B2
A1
C1
C2


A_1
B_1
B_2
C_1
C_2

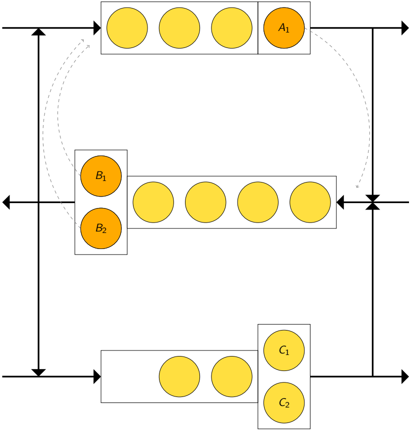

B_1
B_2
A_1
C_1
C_2



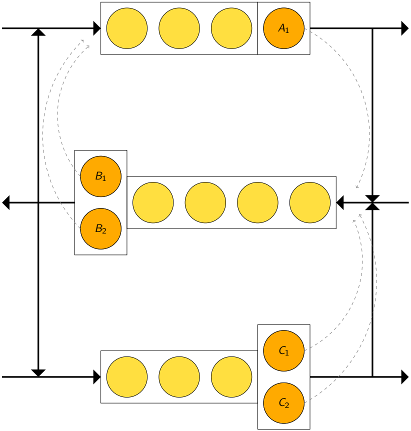
A_1
B_1
B_2
C_1
C_2
B_1
B_2
A_1
C_1
C_2

Types of Deadlock

Transient Deadlock

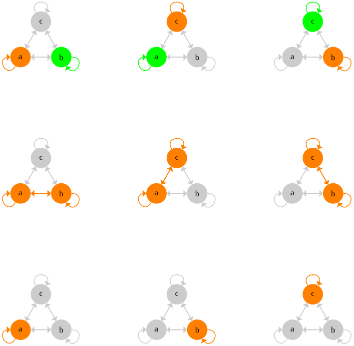


Absorbing Deadlock

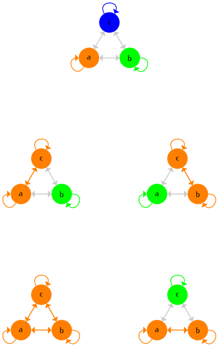


Deadlock Configurations

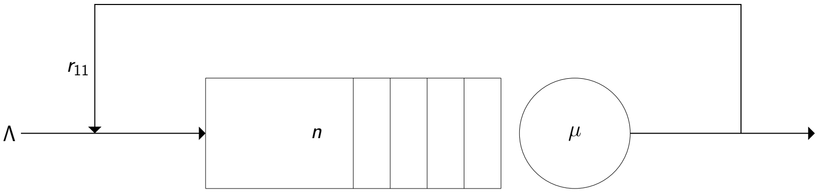
Transient Deadlock



Absorbing Deadlock



Markovian Model of Deadlock



(i)

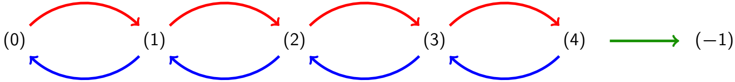
$$S = \{i \in \mathbb{N} | 0 \leq i \leq n+1\} \cup \{-1\}$$

Define $\delta = i_2 - i_1$

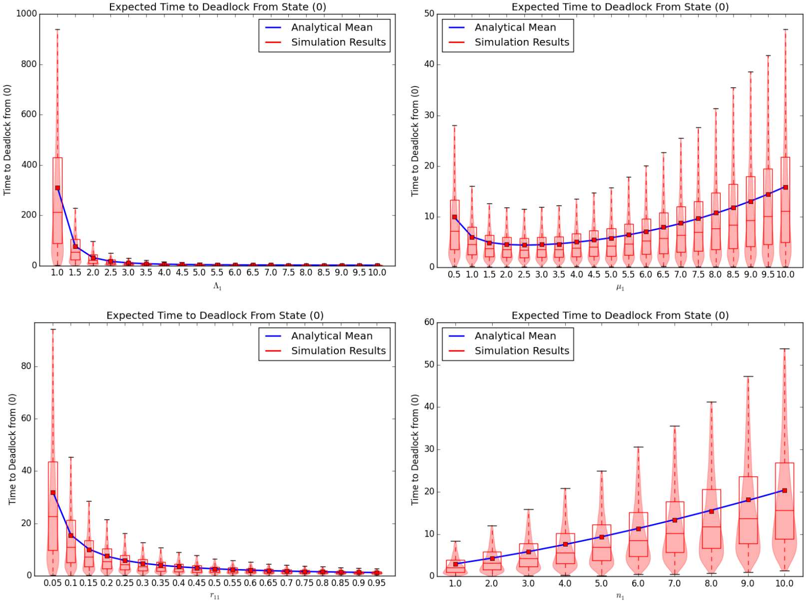
$$q_{i_1,i_2} = \begin{cases} \left.\begin{matrix} \Lambda & \text{if } i < n+1 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = 1 \\ (1 - r_{11})\mu & \text{if } \delta = -1 \\ 0 & \text{otherwise} \end{cases}$$

$$q_{i,-1} = \begin{cases} r_{11}\mu & \text{if } i = n+1 \\ 0 & \text{otherwise} \end{cases}$$

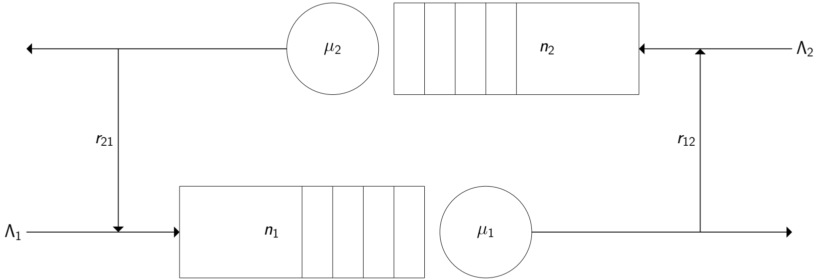
$$q_{-1,s} = 0$$


(0)
(1)
(2)
(3)
(4)
(−1)

Times to Deadlock



Markovian Model of Deadlock



$$(i, j)$$

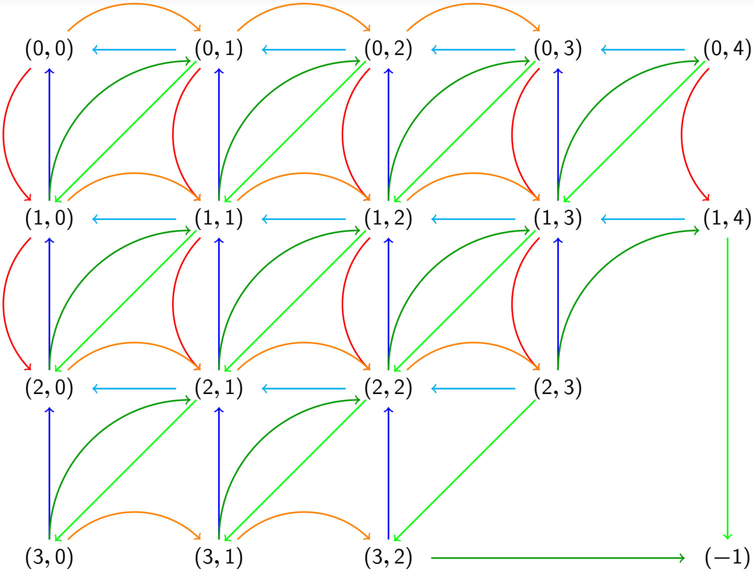
$$S = \{(i,j) \in \mathbb{N}^{(n_1+2\times n_2+2)} | 0 \leq i+j \leq n_1+n_2+2\} \cup \{(-1)\}$$

Define $\delta = (i_2, j_2) - (i_1, j_1)$

$$q_{(i_1,j_1),(i_2,j_2)} = \begin{cases} \left.\begin{matrix} \Lambda_1 & \text{if } i_1 \leq n_1 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (1,0) \\ \left.\begin{matrix} \Lambda_2 & \text{if } j_1 \leq n_2 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (0,1) \\ \left.\begin{matrix} (1-r_{12})\mu_1 & \text{if } j_1 < n_2+2 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (-1,0) \\ \left.\begin{matrix} (1-r_{21})\mu_2 & \text{if } i_1 < n_1+2 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (0,-1) \\ \left.\begin{matrix} r_{12}\mu_1 & \text{if } j_1 < n_2+2 \text{ and } (i_1,j_1) \neq (n_1+2, n_2) \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (-1,1) \\ \left.\begin{matrix} r_{21}\mu_2 & \text{if } i_1 < n_1+2 \text{ and } (i_1,j_1) \neq (n_1, n_2+2) \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (1,-1) \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1,j_1),(-1)} = \begin{cases} r_{21}\mu_2 & \text{if } (i,j) = (n_1, n_2+2) \\ r_{12}\mu_1 & \text{if } (i,j) = (n_1+2, n_2) \\ 0 & \text{otherwise} \end{cases}$$

$$q_{-1,s} = 0$$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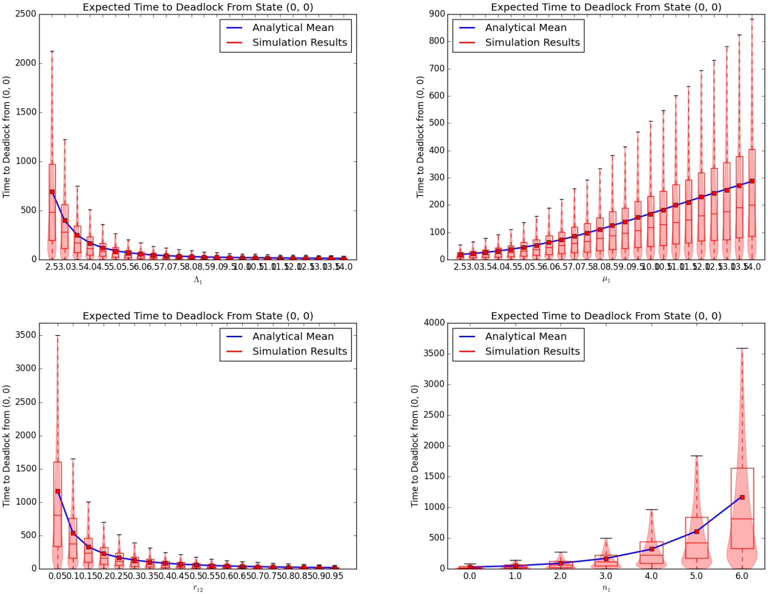
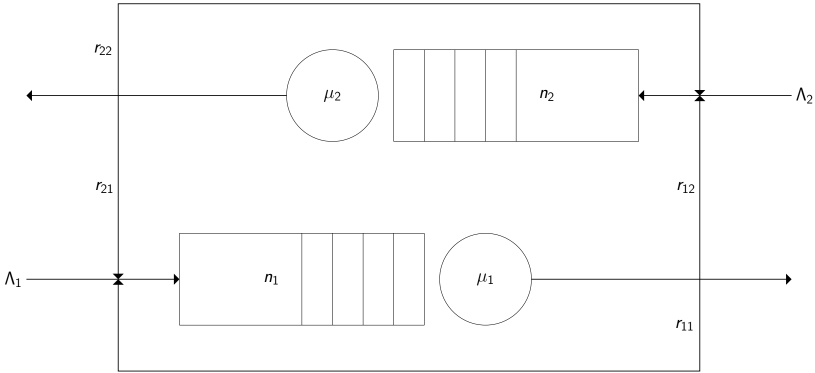
(0, 1)
(0, 2)
(0, 3)
(0, 4)
(1, 0)
(1, 1)
(1, 2)
(1, 3)
(1, 4)
(2, 0)
(2, 1)
(2, 2)
(2, 3)
(3, 0)
(3, 1)
(3, 2)
(−1)

Times to Deadlock



Markovian Model of Deadlock



$$(i, j)$$

$$S = \{(i,j) \in \mathbb{N}^{(n_1+2\times n_2+2)} | 0 \le i+j \le n_1+n_2+2\} \cup \{(-1)\}$$

Define $\delta = (i_2, j_2) - (i_1, j_1)$

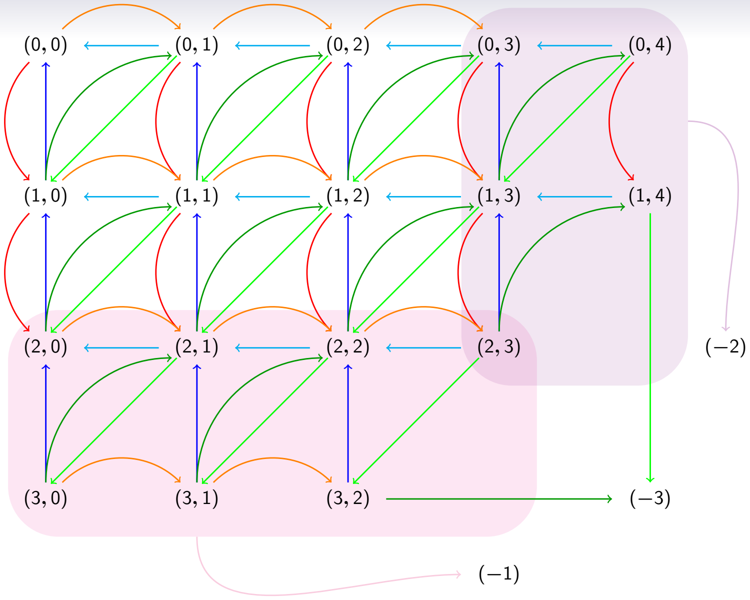
$$q_{(i_1,j_1),(i_2,j_2)} = \begin{cases} \left.\begin{matrix} \Lambda_1 & \text{if } i_1 \le n_1 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (1,0) \\ \left.\begin{matrix} \Lambda_2 & \text{if } j_1 \le n_2 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (0,1) \\ \left.\begin{matrix} (1-r_{12})\mu_1 & \text{if } j_1 < n_2+2 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (-1,0) \\ \left.\begin{matrix} (1-r_{21})\mu_2 & \text{if } i_1 < n_1+2 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (0,-1) \\ \left.\begin{matrix} r_{12}\mu_1 & \text{if } j_1 < n_2+2 \text{ and } (i_1,j_1) \neq (n_1+2, n_2) \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (-1,1) \\ \left.\begin{matrix} r_{21}\mu_2 & \text{if } i_1 < n_1+2 \text{ and } (i_1,j_1) \neq (n_1, n_2+2) \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (1,-1) \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1,j_1),(-1)} = \begin{cases} r_{11}\mu_1 & \text{if } i > n_1 \text{ and } j < n_2+2 \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1,j_1),(-2)} = \begin{cases} r_{22}\mu_2 & \text{if } j > n_2 \text{ and } i < n_1+2 \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1,j_1),(-3)} = \begin{cases} r_{21}\mu_2 & \text{if } (i,j) = (n_1, n_2+2) \\ r_{12}\mu_1 & \text{if } (i,j) = (n_1+2, n_2) \\ 0 & \text{otherwise} \end{cases}$$

$$q_{-1,s} = q_{-2,s} = q_{-3,s} = 0$$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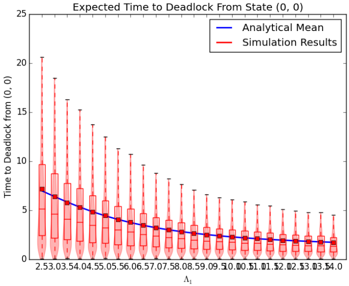
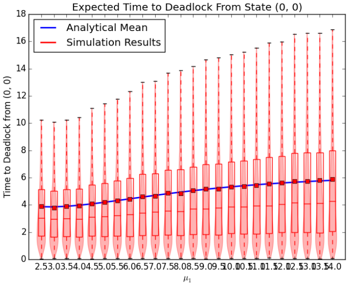
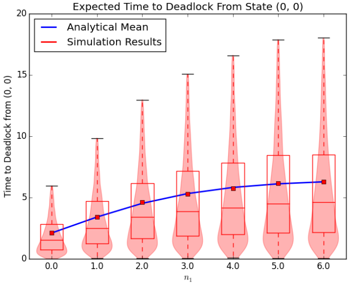
(0, 1)
(0, 2)
(0, 3)
(0, 4)
(1, 0)
(1, 1)
(1, 2)
(1, 3)
(1, 4)
(2, 0)
(2, 1)
(2, 2)
(2, 3)
(−2)
(3, 0)
(3, 1)
(3, 2)
(−3)
(−1)

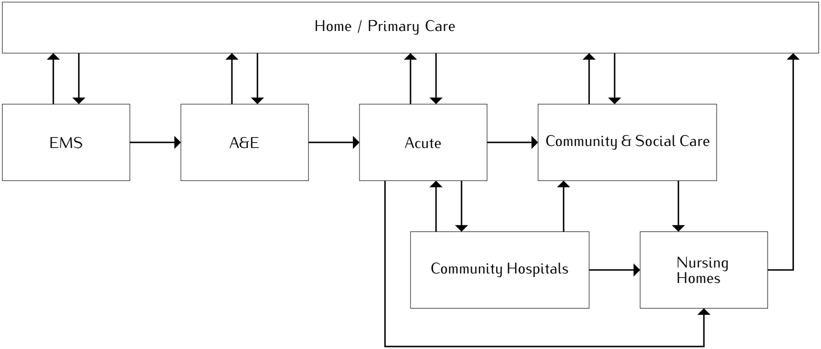
Times to Deadlock



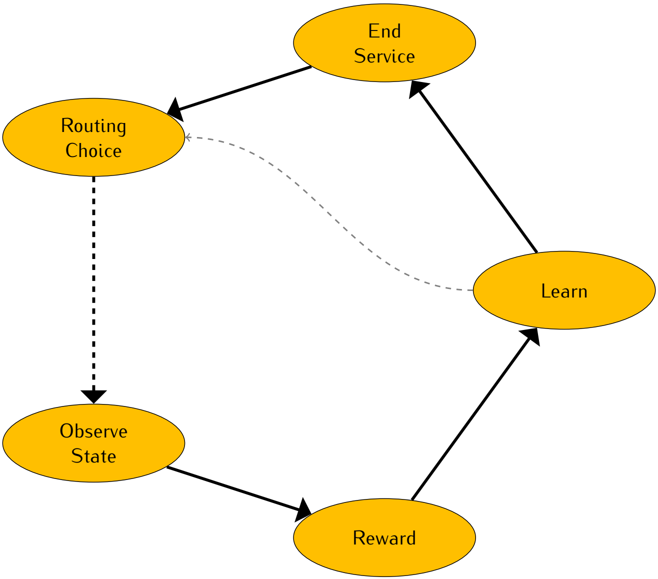




Reinforcement Learning



Reinforcement Learning



Reinforcement Learning

$$Q(s,a) \leftarrow Q(s,a) + \alpha[r + \gamma \max_{a'} Q(s',a') - Q(s,a)]$$

s_n	a_1	a_2	a_3
$n=0$	10.2	3.3	−2.1
$n=1$	9.4	2.1	−0.7
$n=2$	9.5	3.8	1.6
$n=3$	8.3	5.4	5.3
$n=4$	3.1	9.2	6.7
$n=5$	4.0	6.1	6.7
$n=6$	0.2	6.3	7.5

Table: Example table of $Q(s,a)$

Diolch - Thank You

https://github.com/geraintpalmer/Presentations

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